Teaching the magnetostatic field: Problems to avoid

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A widely recognized problem that physics teachers encounter is the difficulty that most students have when solving problems related to magnetic field line distributions in the presence of hard- and softmagnetic materials. Two causes of these difficulties are identified: (1) The fact that the hysteresis is introduced as the typical behavior of ferromagnetic materials; (2) the fact that the magnetic field strength **H** is almost absent in the electromagnetism course. Proposals to remedy the student's problems are (a) to introduce four idealized magnetic materials, i.e., unmagnetic, hardmagnetic, softmagnetic, and superconducting materials; and (b) to use the magnetic field strength **H**, and not the magnetic induction **B**, when discussing problems of magnetostatics.

I. INTRODUCTION

Ask a student who has already completed an electrodynamics course to sketch the magnetic field lines for an arrangement of magnetic poles and a piece of soft iron like that of Fig. 1. Most likely, the student will be helpless. If you had asked him or her to sketch the electric field lines of the arrangement of Fig. 2, he or she would have drawn something like Fig. 3: The qualitatively correct field line distribution. Strangely enough, the problem of Fig. 1 is essentially the same as that of Fig. 2. The mathematical structures of both these problems are identical. Accordingly, finding the solution to one problem should not be any more difficult than finding the solution to the other.

I gave a test to a group of physics students who were between their fourth and sixth semester. All of these students had attended a one-semester course of experimental electrodynamics, a one-semester course of theoretical electrodynamics, and a lab on electromagnetism. I had divided the students in two groups. Each group had to solve one problem of electrostatics and one of magnetostatics. The first group's electrostatics problem had the same structure as the magnetostatics problem of the second group. Furthermore, the magnetostatics problem of the first group was also the counterpart of the electrostatics problem of the second group. The result of the test was obvious: The magnetostatic versions of both problems were solved correctly by 22% of the students. 80% of the students answered the electrostatics problems correctly. This result is all the more remarkable because everybody has a lot of practical experience in dealing with permanent magnets but almost none in dealing with electrically charged bodies. Apparently, the electrodynamics courses have failed in achieving one of their objectives.

In the present article I shall show that the difficulties students encounter when trying to solve problems of magnetostatics have two causes. One is the way we treat magnetic properties of materials; in particular, the fact that hysteresis of these materials is placed so much in the foreground. I propose to distinguish between four classes of idealized materials in an introductory course about electromagnetism: nonmagnetizable materials, ideal magnetically "hard" materials, ideal magnetically "soft" materials, and ideal diamagnetic materials. By magnetically soft materials, or softmagnetic materials for short, we mean materials with a very low remanent field and a very high permeability. Magnetically hard or hardmagnetic materials are those with high coercivity. This proposal will be discussed in Sec. II.

The second cause of the student's difficulties with magnetostatics is due to the predominance of the magnetic induction **B** over the magnetic field strength **H** in the major-

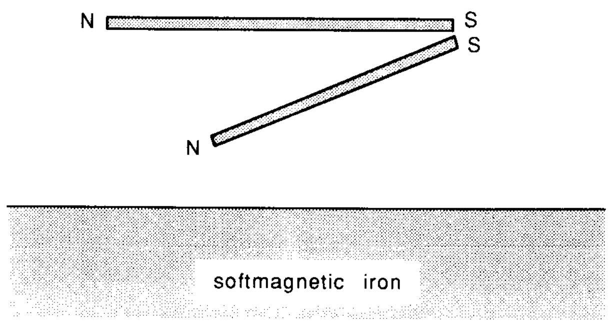

Fig. 1. What is the shape of the magnetic field lines?

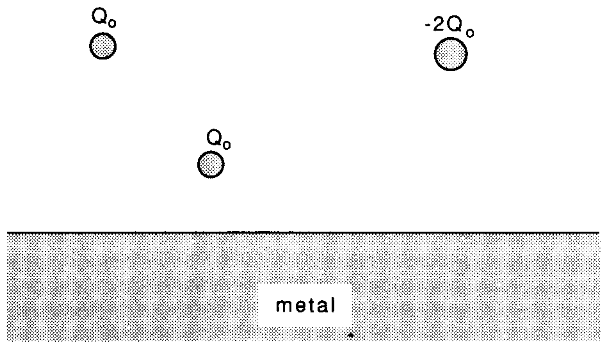

Fig. 2. What is the shape of the electric field lines?

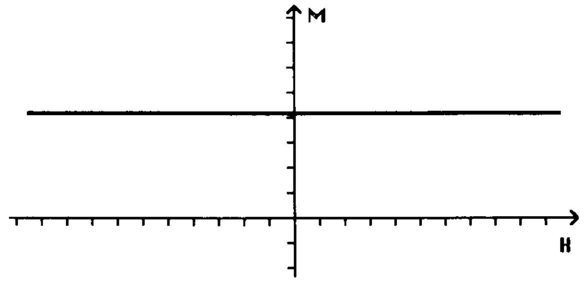

Fig. 4. In an ideal hardmagnetic material, the magnetization M is independent of the magnetic field strength H.

ity of courses and textbooks. I suggest giving more emphasis to the magnetic field strength **H** as long as we deal with magnetostatics. This proposal will be discussed in Sec. III.

Applying the strategy proposed in Sec. III several examples of magnetostatics problems, along with their electrostatic counterparts, will be discussed in Sec. IV. Notice that this article deals with macroscopic phenomena of magnetostatics.

II. FOUR IDEALIZED MAGNETIC MATERIALS

A typical way of introducing magnetic properties of matter is to begin with para- and diamagnetism. Thereafter, ferromagnetism is discussed and hysteresis is introduced as a typical behavior of ferromagnetic materials.[1-6]

I would like to criticize this method of proceeding in two respects—at least as long as an introductory physics course in concerned.

My first criticism concerns the fact that one begins with para- and diamagnetism, i.e., effects of the order of 10^{-4} to 10^{-6}. In an introductory course we would be better off beginning with or even limiting ourselves to strong effects, in this case to ferromagnetic materials. This procedure is completely legitimate and is absolutely customary in other domains of physics. For instance, think of mechanics where we naturally treat a massive iron block as a rigid body, and neglect both its elasticity and its viscosity.

My second objection refers to the way one deals with ferromagnetic materials. Instead of introducing hysteresis as a phenomenon that is characteristic for ferromagnetic materials, it might be better to begin the teaching of ferromagnetism with perfect hardmagnetic and with perfect softmagnetic materials. Today one is able to manufacture very good hardmagnetic as well as softmagnetic materials; therefore, in many technical applications hysteresis does not play an important role.

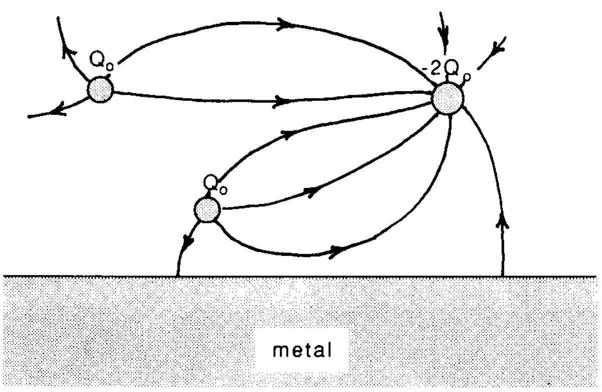

Fig. 3. Qualitatively correct sketch of the field lines of the problem of Fig. 2.

The following is my proposal. We introduce four magnetically different materials. Although these are idealizations, most real materials that are technically employed are very good approximations of these idealized materials.

Our first magnetically perfect material is the ideal hardmagnetic material. It is the material one needs to manufacture permanent magnets. In the factory, a permanent magnet receives the desired distribution of magnetization and, if it is a good magnet, it retains this magnetization regardless of whatever external magnetic fields it is exposed to. The **M–H** characteristic of an ideal hardmagnetic material is shown in Fig. 4.

Of course, one can change the magnetization by "brute force," i.e., by applying fields of high field strengths. This, however, should not hinder us from introducing a material such as that characterized by the diagram in Fig. 4 as an ideal hardmagnetic material. After all, we proceed in the very same manner in other parts of the physics course. Although Hooke's law, Ohm's law, or the linear law for thermal expansion are only valid in a limited domain of the independent variable we do not hesitate to introduce them in our physics course. Only in this way can we achieve order in the diversity of phenomena. Naturally, we have to be careful not to exceed the domain of validity of these laws. However, as long as we do not overstretch a spring, Hooke's law is valid and as long as we do not expose a permanent magnet to an excessive magnetic field, its magnetization remains constant. Moreover, springs also exhibit hysteresis and the microscopic origin of this phenomenon is most interesting. Nevertheless, at the beginning of our mechanics course, we limit ourselves to elastic springs and may treat the mechanical hysteresis when we discuss dislocations in the framework of a course about solid-state physics.

The second perfect magnetic material is the ideal softmagnetic material. The behavior of such a material is as follows: When placed within a magnetic field it magnetizes itself in such a way that the magnetic field strength **H** in its interior remains zero. In an **M–H** diagram it is characterized by a vertical line through the origin, i.e., a line that coincides with the **M** axis.

While the material is being magnetized, poles are forming on its surface. The field strength vectors on the outside of the material, immediately on its surface, are perpendicu-

Table I. Four types of idealized magnetic materials.

Nonmagnetic material:	The field penetrates as if the material were not there.
Hardmagnetic material:	The magnetization is unalterable; an external field penetrates as if the material were not there.
Softmagnetic material:	Pushes an external field away; forms poles on its surface; **H** vectors are perpendicular to its surface.
Superconductor:	Pushes an external field away; forms electric currents on its surface; **H** vectors are parallel to its surface.

lar to the surface. Any metal would be the electrostatic counterpart of these materials. If a metal is placed within an electric field, charges are displaced within the material in such a way that the electric field strength within its interior remains zero.

The comments that were made for hardmagnetic materials can also be made for softmagnetic materials. As long as the magnetic field strengths are not too high, certain materials available today behave like perfect softmagnetic materials to a great degree. The real material gets saturated and the magnetic field penetrates into its interior only if it gets exposed to very strong magnetic fields.

It is understandable that hysteresis plays an important role in older textbooks because the magnetic materials one was able to manufacture were still far from ideal. However, in my opinion, it is completely justified to introduce ideal materials today in a physics course.

Of course, I do not recommend eliminating hysteresis from the curriculum. We must not forget also that the phenomenon of hysteresis has important technical applications: All magnetic data storage devices are based upon this effect. Yet, it is conceptually easier first to introduce the simple cases and then to treat the hysteresis, just as in mechanics we begin with massless springs and only later may introduce springs with nonzero mass. Likewise, in thermodynamics, we also discuss the ideal gas first and the real gas later.

With regard to the magnetic properties we can identify two more ideal types of materials. One of them is, of course, the magnetically inert material: Materials that are not magnetic and not magnetizable such as copper, glass, wood, or plastic. As stated before, in a first approach we neglect para- and diamagnetism.

Finally, the superconductors belong to our fourth class and are often referred to as perfect diamagnets. These perfect diamagnets have something in common with the perfect softmagnets: They don't allow a magnetic field to penetrate into their interior. The way they do so, however, is not the same. Whereas softmagnetic materials form magnetic poles on their surfaces, superconductors form electric currents. Outside, near the surface, the **H** field strength vectors are parallel to this surface, since there are no magnetic poles on the surface, i.e., sources for the **H** field strength.

Table I summarizes the properties of our four basic types of magnetic materials.

The smaller effects of para- and diamagnetism are discussed later (just as in electrostatics, dielectric materials are introduced at the end): a linear relationship between **M** and **H**. These effects, however, are not the concern of the present article.

III. MAGNETIC FIELD STRENGTH VERSUS MAGNETIC INDUCTION

In order to describe magnetism in matter three different vector quantities can be used: The magnetic induction **B**, the magnetic field strength **H**, and the magnetization **M**. These quantities are related by

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}). \tag{1}$$

The validity of this relation means that only two of the three quantities **B**, **H**, and **M** are independent, i.e., one quantity can be derived from the other two.

Whereas in the classical texts of J. C. Maxwell or J. J. Thomson the vector fields **B** and **H** are treated on an equal footing today in the majority of textbooks (see, for instance, Refs. 7–12) much more emphasis is given to the **B** field, since it is considered to be more fundamental. (I have found only one contemporary textbook in which **H** is preferred to **B**[13]). Indeed, it is better to use **B** when treating electrodynamics on a microscopic scale. Moreover, it is **B** that is responsible for the Lorentz force and, finally, the phenomenon of electromagnetic induction is certainly more transparent when discussed in terms of **B** instead of **H**. However, this custom of preferring **B** over **H** had the consequence that the advantages of operating with **H**, well-known in former times, are now almost forgotten. In order to recognize the advantages of operating with **H** instead of **B**, it suffices to look for the reasons why it is so easy to operate with **E** in electrostatics. The problem of Fig. 2 belongs to a class of problems that can be solved qualitatively by means of a few simple rules. In these problems a distribution of fixed charges and a distribution of electric conductors is supplied. Indisputably, such a problem can be solved by applying one of Maxwell's equations

$$\operatorname{div} \mathbf{E} = \rho/\epsilon_0, \tag{2}$$

and we will admit that this equation is known by our students. Nevertheless, normally a student does not go back to the origin, i.e., to Maxwell's equations. Instead, he or she operates with a set of rules that are easier to manipulate. These rules are a consequence of Eq. (2) and of the fact that electric charges can move freely within electric conductors. The following is a list of the above-mentioned rules.

(1) Electric field lines begin on positive and end on negative electric charges. The number of field lines that begin or end on a charge is proportional to the absolute value of this charge.

(2) Electric field lines never cross.

(3) In a vacuum the direction of an electric field line does not change abruptly.

(4) Inside an electric conductor there are no electric field lines.

(5) At the external surface of a conductor the field lines are perpendicular to this surface.

There are some more rules if one allows for dielectric materials. However, we will limit ourselves to problems without dielectrics.

Now, for the magnetic field lines, i.e., the lines of the **H** field strength, a set of rules is valid which is analogous to the aforementioned rules for electric fields. Again, we start with one of Maxwell's equations:

$$\mathrm{div}\,\mathbf{B} = 0.$$

Using Eq. (1), we get

$$\mathrm{div}\,\mathbf{H} = -\mathrm{div}\,\mathbf{M}, \tag{3}$$

and introducing a magnetic pole charge density ρ_m by

$$\rho_m = -\mathrm{div}\,\mathbf{M},$$

Eq. (3) can be written

$$\mathrm{div}\,\mathbf{H} = \rho_m/\mu_0, \tag{4}$$

which has the same mathematical structure as Eq. (2). ρ_m quantitatively describes the distribution of the magnetic pole charge. Our rules for magnetic field lines follow from Eq. (4) and from the fact that in softmagnetic materials magnetic poles can move around freely:

(1′) Magnetic field lines begin on positive (north) and end on negative (south) magnetic poles. The number of field lines that begin or end on an magnetic pole is proportional to the absolute value of the pole charge.

(2′) Magnetic field lines never cross.

(3′) In a vacuum the direction of a magnetic field line does not change abruptly.

(4′) Inside a softmagnetic material there are no magnetic field lines.

(5′) At the external surface of a softmagnetic material the field lines are perpendicular to this surface.

By comparing Eq. (2) with Eq. (4) or rules (1) to (5) with rules (1′) to (5′) one sees that the solutions to the problems of Figs. 1 and 2 are identical: The magnetic field lines of Fig. 1 have the same form as the electric field lines of Fig. 2. Hence, we have shown that the solution of the magnetostatic problem is just as easy as the solution of the electrostatic problem.

If one is interested in the **B** field lines these can be obtained, again qualitatively, by combining the **H** field distribution and the magnetization, which is either supplied or easy to construct in our problems.

What is the reason for the problem students have if they try to draw the **B** field lines from the beginning? Isn't there an analogous set of rules for **B** field lines? Of course there is. Three of the above rules for the **H** field strength are valid for the **B** field strength as well: Rules (2′), (3′), and (5′). Rule (1′) has to be replaced with the following rule:

(1″) **B** field lines are always closed.

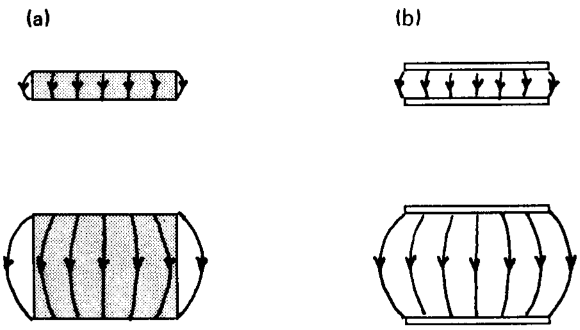

Fig. 5. (a) Magnetic field lines of two cylindrical pieces of a homogeneously magnetized material. The axis of the cylinders are vertical in the figure. The upper cylinder is shorter in the direction of the cylinder's axis than the lower one. (b) The electric field lines belonging to pairs of circular plates with equal charges of opposite sign have the same shape as the magnetic field lines in (a).

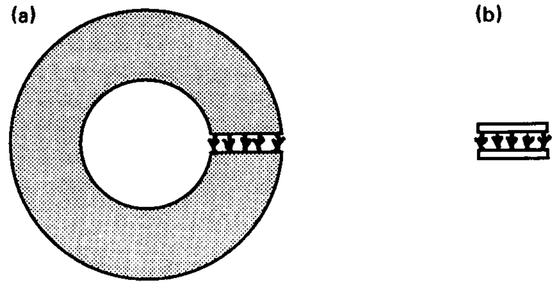

Fig. 6. (a) Ring magnet with circular cross section. The poles are the two surfaces of the slit. The H field lines have the same shape as for the upper magnet of Fig. 5(a). (b) The electrostatic counterpart is the same as that of the upper magnet in Fig. 5(a).

There is no rule corresponding to rule (4′).

However, rule (1″) is far less useful for the construction of **B** field lines than rules (1′) and (4′) are for the construction of **H** field lines. While rules (1′) and (4′) relate the field lines to the magnetic poles, i.e., the divergence of the magnetization field, rule (1″) does not.

Moreover, the analogy between **E** and **H** is advantageous in other domains of electromagnetism, for instance, for the calculation of Poynting vector distributions[14] and of mechanical stress distributions.[15] It is limited, of course, by the fact that nature did not realize—as far as we know today—isolated magnetic charges. As a consequence, there are no true magnetic currents.

Finally, we add two more magnetic rules, (6′) and (7′), which do not have an electric counterpart. Just as (4′) and (5′) are rules valid for softmagnetic materials, (6′) and (7′) are rules about superconducting materials:

(6′) Inside a superconducting material there are no magnetic field lines.

(7′) At the external surface of a superconductor the field lines are parallel to the surface.

IV. EXAMPLES

In this section I discuss some examples of **H** field distributions. In each of these examples, one finds the **H** field lines that belong to a given distribution of magnetic poles, softmagnetic materials, and superconductors by applying rules (1′) to (7′) of Sec. III. For each magnetic problem we have sketched the analogous electric problem with its solution, except for the last two since the counterpart of an electric superconductor is not realized in nature. All the field lines are sketched by hand.

(1) *Cylindrical permanent magnets, Fig. 5.* The material

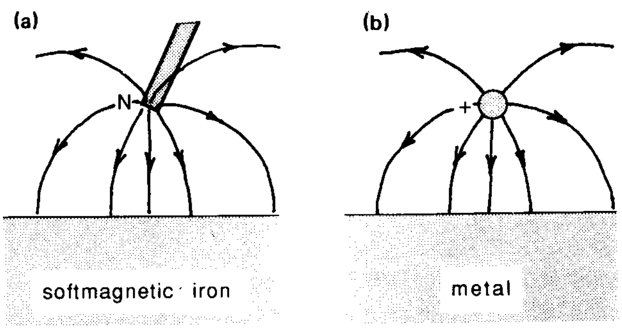

Fig. 7. (a) Magnetic north pole near to a plane softmagnetic surface. (b) Positive electric charge near to a plane metal surface.

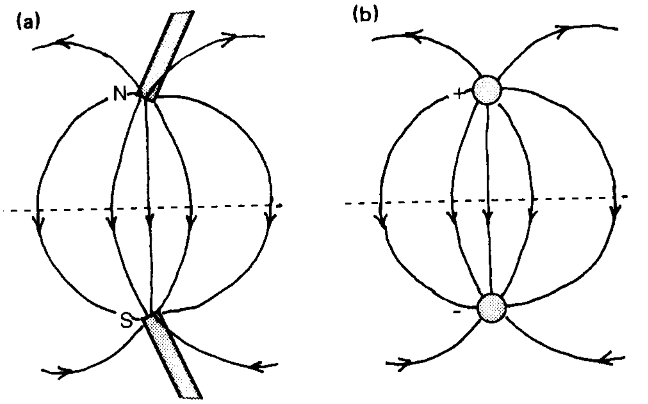

Fig. 8. (a) The softmagnetic plate of Fig. 7(a) has been replaced with an "image" south pole. (b) The metal plate of Fig. 7(b) has been replaced with a negative "image" charge.

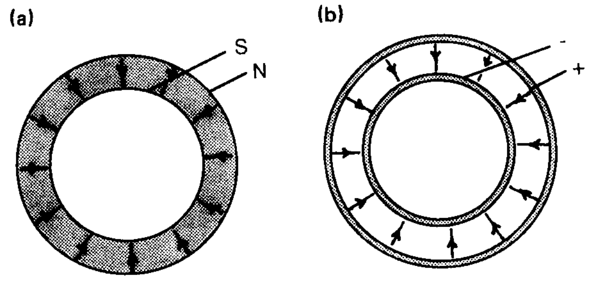

Fig. 10. (a) Magnetized spherical layer. The inner side carries south pole charge, the outer side north pole charge. (b) Electrostatic counterpart: two concentric metal spheres that carry equal and opposite charge.

is homogeneously magnetized. The magnetization is parallel to the cylinder axis. The electric counterpart consists in two circular parallel plates that carry equal and opposite charges. The charge per surface area is constant on each plate.

(2) *Ring-shaped permanent magnet with a slit, Fig. 6.* The cross section of the ring is circular. The **H** field is identical with that of the flat cylindric magnet of Fig. 5. For a ring magnet without a slit, the magnetic field strength is zero everywhere.

(3) *Magnetic pole near a plane softmagnetic surface, Fig. 7.* The field is identical with the upper half of the field of Fig. 8. To obtain the arrangement of Fig. 8 the pole distribution of Fig. 7 has been mirrored on the surface of the softmagnetic body and the softmagnetic body has been replaced by the "image pole."[16]

(4) *Rectangular bar magnet with soft iron yoke, Fig. 9.* The permanent bar magnet induces magnetic poles in the soft iron parts. The field of the poles two and three of the permanent magnet is compensated by the field of the induced poles 1 and 4. There remains the field of the induced poles 5 and 6. The electric counterpart consists of two oppositely charged plates 2 and 3. These plates induce opposite charges in plates 1 and 4, respectively. The field of charges 2 and 3 is compensated by the field of the induced charges 1 and 4. There remains the field of the induced charges 5 and 6.

(5) *Magnetized spherical layer, Fig. 10.* The outer surface carries north pole charge, the inner surface south pole charge. The corresponding electrostatics problem is the spherical condenser, familiar to every physics student. The space inside and outside the layer is free of field. Only within the layer the **H** or **E** field strengths, respectively, are different from zero.

(6) *Magnetic pole near to a plane superconductor, Fig. 11(a).* The field is identical with the upper half of the field of Fig. 11(b). To obtain the arrangement of Fig. 11(b) the pole distribution of Fig. 11(a) has been mirrored on the surface of the superconductor and the superconductor has been replaced by the "image pole." By means of this second pole the boundary condition represented by the superconductor in Fig. 11(a) has been simulated, namely, the fact that the field lines at the superconductor surface are parallel to this surface. Contrary to the familiar "method of images" or the corresponding magnetic rule (see our example 3 or Jackson[16]), the sign of the charge of the image pole is the same as that of the original pole charge.

(7) *Bar magnet in superconducting tube, Fig. 12.* A short cylindric bar magnet fits exactly into a long superconducting tube. **H** field lines are only within the bar magnet. To be in agreement with rules (6′) and (7′) the **H** field lines have to be parallel to the magnet's axis. There are no field lines on the left side of the north pole and on the right side of the south pole, because here the contributions of the north and of the south pole areas compensate each other.

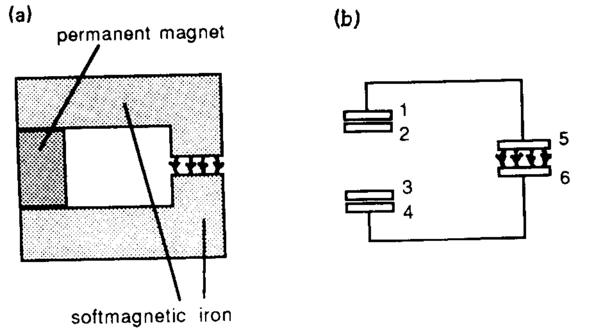

Fig. 9. (a) Ring magnet consisting of a rectangular bar magnet and two L-shaped soft iron pieces. (b) The electric counterpart consists of three capacitors. The capacitors 1–2 and 3–4 have zero plate separation.

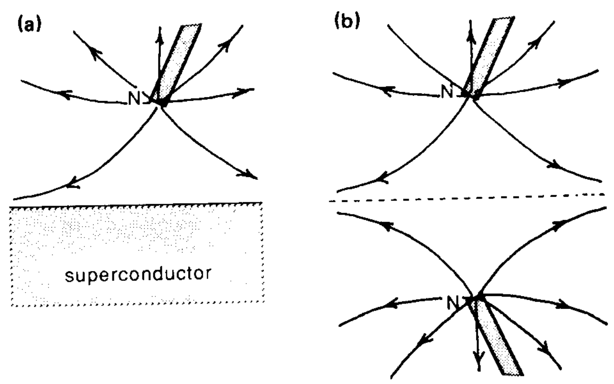

Fig. 11. (a) Magnetic north pole near to a plane superconducting surface. (b) The superconducting plate has been replaced with an "image" north pole.

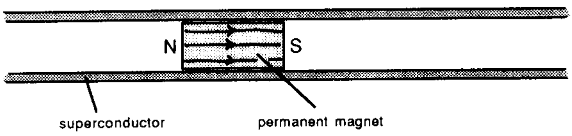

Fig. 12. Cylindrical bar magnet that fits exactly into a long superconducting tube.

V. CONCLUSION

Several proposals have been made for rearranging an introductory course on electromagnetism in order to avoid difficulties which the students have when solving magnetostatics problems. These proposals can be summarized as follows:

(1) Do not begin magnetism in matter with para- and diamagnetism, rather begin with ferromagnets and superconductors. Para- and diamagnetism are effects of the order of 10^{-4} to 10^{-6} and can be neglected in a first approach to magnetism.

(2) When treating ferromagnetic materials, do not begin with hysteresis but with idealized softmagnetic and hardmagnetic materials. There are four pure ideal cases of magnetic properties of matter. The corresponding materials are: unmagnetic, softmagnetic, hardmagnetic, and superconducting materials.

(3) When treating hard- and softmagnetic materials, operate with the vectors **H** and **M** and not with **B** and **M**.

[1] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1963), Vol. II, Chaps. 34–36.

[2] E. M. Purcell, *Berkeley Physics Course: Electricity and Magnetism* (McGraw-Hill, New York, 1965), Chap. 10, pp. 352–399.

[3] D. Halliday and R. Resnick, *Physics* (Wiley, New York, 1977), Pt. 2, Chap. 37, pp. 820–825.

[4] J. R. Reitz and F. J. Milford, *Foundations of Electromagnetic Theory* (Addison-Wesley, Reading, MA, 1960), pp. 193–195.

[5] W. R. Smythe, *Static and Dynamic Electricity* (McGraw-Hill, New York, 1950), p. 421.

[6] R. S. Elliott, *Electromagnetics* (McGraw-Hill, New York, 1966), pp. 416–436.

[7] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), p. 188.

[8] Reference 1, Chap. 36.

[9] Reference 2, p. 390.

[10] M. Alonso and E. J. Finn, *Fundamental University Physics* (Addison-Wesley, Reading, MA, 1967), Vol. II, Chap. 16, p. 603.

[11] Reference 3, p. 833.

[12] M. Schwartz, *Principles of Electrodynamics* (McGraw-Hill, New York, 1972), p. 155.

[13] W. Macke, *Elektromagnetische Felder* (Akademische Verlagsgesellschaft, Leipzig, 1960), Chap. 34, p. 135.

[14] F. Herrmann and G. B. Schmid, "The Poynting vector field and the energy flow within a transformer," Am. J. Phys. **54**, 528–531 (1986).

[15] F. Herrmann, "Energy density and stress: A new approach to teaching electromagnetism," Am. J. Phys. **57**, 707–714 (1989).

[16] Reference 7, problem 5.9, p. 207.